OCR
RECOGNISING ACHIEVEMENT

Tuesday 18 June 2013 – Morning

A2 GCE MATHEMATICS

4724/01 Core Mathematics 4

QUESTION PAPER

Candidates answer on the Printed Answer Book.

OCR supplied materials:
- Printed Answer Book 4724/01
- List of Formulae (MF1)

Other materials required:
- Scientific or graphical calculator

Duration: 1 hour 30 minutes

INSTRUCTIONS TO CANDIDATES

These instructions are the same on the Printed Answer Book and the Question Paper.

- The Question Paper will be found in the centre of the Printed Answer Book.
- Write your name, centre number and candidate number in the spaces provided on the Printed Answer Book. Please write clearly and in capital letters.
- **Write your answer to each question in the space provided in the Printed Answer Book**. Additional paper may be used if necessary but you must clearly show your candidate number, centre number and question number(s).
- Use black ink. HB pencil may be used for graphs and diagrams only.
- Answer **all** the questions.
- Read each question carefully. Make sure you know what you have to do before starting your answer.
- Do **not** write in the bar codes.
- You are permitted to use a scientific or graphical calculator in this paper.
- Give non-exact numerical answers correct to 3 significant figures unless a different degree of accuracy is specified in the question or is clearly appropriate.

INFORMATION FOR CANDIDATES

This information is the same on the Printed Answer Book and the Question Paper.

- The number of marks is given in brackets **[ ]** at the end of each question or part question on the Question Paper.
- **You are reminded of the need for clear presentation in your answers.**
- The total number of marks for this paper is **72**.
- The Printed Answer Book consists of **12** pages. The Question Paper consists of **4** pages. Any blank pages are indicated.

INSTRUCTION TO EXAMS OFFICER / INVIGILATOR

- Do not send this Question Paper for marking; it should be retained in the centre or recycled. Please contact OCR Copyright should you wish to re-use this document.

1 Express $\dfrac{(x-7)(x-2)}{(x+2)(x-1)^2}$ in partial fractions. **[5]**

2 Find $\int x^8 \ln(3x)\,\mathrm{d}x$. **[5]**

3 Determine whether the lines whose equations are

$$\mathbf{r} = (1+2\lambda)\mathbf{i} - \lambda\mathbf{j} + (3+5\lambda)\mathbf{k} \quad \text{and} \quad \mathbf{r} = (\mu-1)\mathbf{i} + (5-\mu)\mathbf{j} + (2-5\mu)\mathbf{k}$$

are parallel, intersect or are skew. **[6]**

4 The equation of a curve is $y = \cos 2x + 2\sin x$. Find $\dfrac{\mathrm{d}y}{\mathrm{d}x}$ and hence find the coordinates of the stationary points on the curve for $0 < x < \pi$. **[6]**

5 **(i)** Show that $\dfrac{1}{1-\tan x} - \dfrac{1}{1+\tan x} \equiv \tan 2x$. **[2]**

(ii) Hence evaluate $\displaystyle\int_{\frac{1}{12}\pi}^{\frac{1}{6}\pi} \left(\frac{1}{1-\tan x} - \frac{1}{1+\tan x}\right)\mathrm{d}x$, giving your answer in the form $a\ln b$. **[5]**

6 Use the substitution $u = 1 + \ln x$ to find $\displaystyle\int \frac{\ln x}{x(1+\ln x)^2}\,\mathrm{d}x$. **[6]**

7 Points A (2, 2, 5), B (1, −1, −4), C (3, 3, 10) and D (8, 6, 3) are the vertices of a pyramid with a triangular base.

(i) Calculate the lengths AB and AC, and the angle BAC. **[4]**

(ii) Show that $\overrightarrow{AD}$ is perpendicular to both $\overrightarrow{AB}$ and $\overrightarrow{AC}$. **[3]**

(iii) Calculate the volume of the pyramid $ABCD$. **[3]**

[The volume of the pyramid is $V = \dfrac{1}{3} \times$ base area $\times$ perpendicular height.]

8 At time t seconds, the radius of a spherical balloon is r cm. The balloon is being inflated so that the rate of increase of its radius is inversely proportional to the square root of its radius. When $t = 5$, $r = 9$ and, at this instant, the radius is increasing at $1.08\,\mathrm{cm\,s^{-1}}$.

(i) Write down a differential equation to model this situation, and solve it to express r in terms of t. **[7]**

(ii) How much air is in the balloon initially? **[2]**

[The volume of a sphere is $V = \frac{4}{3}\pi r^3$.]

9 A curve has parametric equations $x = \frac{1}{t} - 1$ and $y = 2t + \frac{1}{t^2}$.

(i) Find $\frac{\mathrm{d}y}{\mathrm{d}x}$ in terms of t, simplifying your answer. **[3]**

(ii) Find the coordinates of the stationary point and, by considering the gradient of the curve on either side of this point, determine its nature. **[4]**

(iii) Find a cartesian equation of the curve. **[2]**

10 **(i)** Show that $\frac{x}{(1-x)^3} \approx x + 3x^2 + 6x^3$ for small values of x. **[2]**

(ii) Use this result, together with a suitable value of x, to obtain a decimal estimate of the value of $\frac{100}{729}$. **[2]**

(iii) Show that $\frac{x}{(1-x)^3} = -\frac{1}{x^2}\left(1 - \frac{1}{x}\right)^{-3}$. Hence find the first three terms of the binomial expansion of $\frac{x}{(1-x)^3}$ in powers of $\frac{1}{x}$. **[4]**

(iv) Comment on the suitability of substituting the same value of x as used in part **(ii)** in the expansion in part **(iii)** to estimate the value of $\frac{100}{729}$. **[1]**

Question	Answer	Marks	Guidance	
1	$\frac{(x-7)(x-2)}{(x+2)(x-1)^2} \equiv \frac{A}{x+2} + \frac{B}{(x-1)} + \frac{C}{(x-1)^2}$ [If no partial fractions seen anywhere, B0]	B1	**SC** $\frac{(x-7)(x-2)}{(x+2)(x-1)^2} \equiv \frac{A}{x+2} + \frac{Bx+C}{(x-1)^2}$ [If no partial fractions seen anywhere, B0]	B1
	$(x-7)(x-2) \equiv A(x-1)^2 + B(x+2)(x-1) + C(x+2)$ [Allow careless minor error but not algebraic method error] or any equiv identity such as $\frac{(x-7)(x-2)}{(x-1)^2} \equiv A + \frac{B(x+2)}{(x-1)} + \frac{C(x+2)}{(x-1)^2}$ (or even the identity on the 1st line), in which values of x are substituted (or cfs compared)	M1	$(x-7)(x-2) \equiv A(x-1)^2 + (Bx+C)(x+2)$ [Allow careless minor error but not algebraic method error] or any equivalent identity (as in previous column) (or even the identity on the 1st line), in which values of x are substituted (or cfs compared)	M1
	$A=4, B=-3, C=2$ or $\frac{4}{x+2} - \frac{3}{x-1} + \frac{2}{(x-1)^2}$ ISW The 3 @ A1 are dep on the used identity being correct. Cover-up: A=4,C=2 score B1,B1; $B=-3$ needs M1, then A1	A1,1,1 **[5]**	$A=4,\ B=-3, C=5$ or $\frac{4}{x+2} + \frac{-3x+5}{(x-1)^2}$	A1 This gives max 3/5 for easier case

Question	Answer	Marks	Guidance	
2	$u = \ln 3x$ and dv or $\frac{dv}{dx} = x^8$	M1	integ by parts as far as f(x)+/– $\int g(x)(dx)$	If difficult to assess, x^8 must be integrated, so look for term in x^9
	$\frac{d}{dx}(\ln 3x) = \frac{1}{x}$ or $\frac{3}{3x}$	B1	stated or clearly used	
	$\frac{x^9}{9}\ln 3x - \int \frac{x^9}{9}$ their $\frac{du}{dx}(dx)$ FT	√A1	i.e. correct understanding of 'by parts'…	..even if ln(3x) incorrectly differentiated
	Indication that $\int kx^8 dx$ is required	M1	i.e. before integrating, product of terms must be taken	The product may already have been indicated on the previous line
	$\frac{x^9}{9}\ln 3x - \frac{x^9}{81}$ or $\frac{1}{9}x^9\left(\ln 3x - \frac{1}{9}\right)$ ISW (+c) <u>cao</u>	A1	$\frac{1}{9}\frac{x^9}{9}$ to be simplif to $\frac{x^9}{81}$; $\frac{3x^9}{243}$ satis	
		[5]		
	<u>If candidate manipulates ln(3x) first of all</u>			
	ln(3x) = ln 3 + ln x	B1		If, however, ln(3x) is said to be ln 3.ln x, then B0 followed by possible M1 A1 A1 in line with alternative solution on LHS, where the 'M' mark is for dealing with $\int x^8 \ln x\, dx$ 'by parts' in the right order and the 2 @ A1 are for correct results.
	$u = \ln x$ and $dv = x^8$	M1	In order to find $\int x^8 \ln x\, dx$:	
	$\frac{x^9}{9}\ln x - \int \frac{x^9}{9}.\frac{1}{x}(dx)$ or better	A1		
	$\frac{x^9}{9}\ln x - \frac{x^9}{81}$	A1		
	Their $\int x^8 \ln x\, dx + \frac{x^9}{9}\ln 3$ (+ c) FT ISW	√A1		

Question			Answer	Marks	Guidance	
3			Set up the 3 relevant equations $1+2\lambda=\mu-1 \qquad -\lambda=5-\mu \qquad 3+5\lambda=2-5\mu$	M1	'M' mark so intention must be clear; minor error(s) only accepted	MR must be consistent; correct version anywhere $\Rightarrow$ not MR
			Attempt to find λ or μ from 2 of the equations & then find μ or λ from any of the 3 equations.	M1	Or find λ, say, from (i)(ii) & then from (ii)(iii) [values shown at next stage] – inconsistency dep*A1 also awarded here	
			$(\lambda,\mu)=$ (3,8) or $(-2\frac{3}{5},2\frac{2}{5})$ or $(-\frac{11}{15},\frac{8}{15})$ or $(3,-3\frac{1}{5})$ or $(-\frac{11}{15},4\frac{4}{15})$ or $(-2\frac{3}{5},-3\frac{1}{5})$ or $(\frac{1}{5},2\frac{2}{5})$ or $(-8\frac{1}{5},8)$ or $(-4\frac{7}{15},\frac{8}{15})$	A1	Accept equivalent proper/improper fractional values or decimal equivalent values	These are all of the solutions obtainable using different combinations of the 3 equations; e.g. using just i & ii or using i & ii to find λ & iii to find μ
			Demonstrate <u>inconsistency</u> i.e. substitute the <u>correct</u> values into a <u>correct</u> equation (but not the immediate last one used)	M1	e.g. after (3,8), subst in iii & write $3+5\times3 \neq 2-5\times8$ or $3+5\times3=2-5\times8 \therefore$ do not intersect	
			State "skew"	A1	Dep on 3 @ M1 + A1	
			(a) Identify direction vectors; (b) state "not identical/same/equal/equiv/multiples" or eval cos(angle) & state $\neq 1(\text{or } -1)$; (c) state "not parallel"	B1	dvs <u>must be identified</u>: $\begin{pmatrix}2\\-1\\5\end{pmatrix}$ and $\begin{pmatrix}1\\-1\\-5\end{pmatrix}$ Accept any vector notation.	
				[6]		

Question			Answer	Marks	Guidance	
4			Use of $\sin 2x = +/-2\sin x\cos x$ or $+/-\cos\left(\frac{\pi}{2}-2x\right)$ *or* $\cos 2x = +/-2\cos^2 x +/-1$ etc $\left(\frac{dy}{dx}=\right)\ -2\sin 2x (\text{or} -4\sin x\cos x);\ +2\cos x$ their $\frac{dy}{dx}=0$ $\left(\frac{\pi}{2},1\right)$; $\left(\frac{\pi}{6},\frac{3}{2}\right)$ and $\left(\frac{5\pi}{6},\frac{3}{2}\right)$	M1 B1,B1 *M1 dep* A1; A1 **[6]**	Seen anywhere in the solution -1(once) for using degrees in an answer instead of radians. If B0 &/or B0 <u>because of sign error</u>, allow A1 to be awarded for $\left(\frac{\pi}{2},1\right)$	<u>SC</u> If A0 but all 3 x-values are correct, award SC A1 <u>SC</u> B2 for 3 ✓answers without working
5	(i)		$\frac{(1+\tan x)-(1-\tan x)}{(1-\tan x)(1+\tan x)}$ $= \frac{2\tan x}{1-\tan^2 x} = \tan 2x$ Answer Given	M1 A1 **[2]**	Combine (or write as 2 separate fractions) using common denominator $\frac{2\tan x}{1-\tan^2 x}$ essential stage N.B. If $\tan x$ changed into $\frac{\sin x}{\cos x}$ before manipulation, apply same principles	Accept with/without brackets in num Accept $1-\tan x.1+\tan x$ in denom A0 for omission of any necessary brackets

Question			Answer	Marks	Guidance	
5	(ii)		$\int \tan 2x \, dx = \lambda \ln(\sec 2x)$ or $\mu \ln(\cos 2x)$ [= F(x)]	M1		
			$\lambda = \frac{1}{2}$ or $\mu = -\frac{1}{2}$	A1		
			their F[$\frac{\pi}{6}$] – their F[$\frac{\pi}{12}$]	M1	dependent on attempt at integration……..	……i.e. not for $\tan\left(\frac{\pi}{3}\right) - \tan\left(\frac{\pi}{6}\right)$
			$\frac{1}{2}\ln 2 - \frac{1}{2}\ln\frac{2}{\sqrt{3}}$ oe	A1	i.e. any correct but probably unsimplified numerical version	
			$\frac{1}{2}\ln\sqrt{3}$ or $\frac{1}{4}\ln 3$ or $\ln 3^{1/4}$ or $\frac{1}{2}\ln\frac{6}{2\sqrt{3}}$ oe ISW	+A1	i.e. any correct version in the form $a \ln b$	
				[5]		

Question			Answer	Marks	Guidance	
6			Find du in terms of dx (or vv) or $\frac{du}{dx}$ or $\frac{dx}{du}$	M1	An attempt - not necessarily accurate	
			Substitute, changing given integral to $\int \frac{u-1}{u^2}(du)$	A1	No evidence of x at this A1 stage	
			Provided of form $\frac{au+b}{u^2}$, either split as $\frac{au}{u^2}+\frac{b}{u^2}\ldots$	M1	or use 'parts' with 'u' = $au+b$, 'dv' = $\frac{1}{u^2}$	
			Integrate as $\ln u+\frac{1}{u}$ or FT as $a\ln u-\frac{b}{u}$ [=F(u)]	√A1	or $-(au+b)\frac{1}{u}+a\ln u$ FT [=G(u)]	
			Re-substitute $u = 1 + \ln x$ in F(u)	M1	Re-substitute $u = 1 + \ln x$ in G(u)	
			$\ln(1+\ln x)+\frac{1}{1+\ln x}$ (+ c) ISW	A1	or $\ln(1+\ln x)-\frac{\ln x}{1+\ln x}$ (+ c) ISW	
				[6]		
7	(i)		**In each part, mark the answers, ignoring the labels**		**To invoke MR, evidence must be clear**	
			$AB = \sqrt{91}$; $AC = \sqrt{27}$ or $3\sqrt{3}$ ISW	B1; B1	9.54 or 9.539392..; 5.2(0) or 5.1961524..	
			Attempting to use $\overrightarrow{AB}.\overrightarrow{AC}=AB.AC\cos\theta$	M1	or $BC^2 = AB^2 + AC^2 - 2AB.AC\cos\theta$	
			angle BAC = 171 (3 sf) or 2.99 (rad) (3 sf) ISW	A1	Final acute answer [8.68 or 0.152] /choice → A0	171 to 171.317 or 2.99
				[4]		
7	(ii)		$6\mathbf{i}+4\mathbf{j}-2\mathbf{k}$ or $-6\mathbf{i}-4\mathbf{j}+2\mathbf{k}$	B1	seen, irrespective of any labelling	
			$6\times(-1)+4\times(-3)-2\times(-9)=0$ $(\therefore$ perpendicular$)$ **AG**	B1	oe using $(6,4,-2)$ or $(-6,-4,2)$ and…	…. $(-1,-3,-9)$ or $(1,3,9)$
			$6\times1+4\times1-2\times5=0$ $(\therefore$ perpendicular$)$ **AG**	B1	oe using $(6,4,-2)$ or $(-6,-4,2)$ and…	… $(1,1,5)$ or $(-1,-1,-5)$
				[3]		
7	(iii)		(AD =) $\sqrt{56}$ or $2\sqrt{14}$ or 7.48… soi	B1		
			area ABC= ½(their)AB × (their)AC × sin(their)BAC	M1	(✓= 3.74… but M mark, not A)	
			$9.3 \le V < 9.35$, $9\frac{1}{3}$ ISW	A1	Accept even if (i) angle given as 8.68…..	i.e. the acute version not accepted in (i)
				[3]		

Question			Answer	Marks	Guidance	
8	(i)		$\frac{dr}{dt} = \frac{k}{\sqrt{r}}$ oe	B2	B1 for $\frac{dr}{dt} =$; B1 for $\frac{k}{\sqrt{r}}$	SR: B1 for $\frac{dr}{dt} \propto \frac{1}{\sqrt{r}}$
			Sep variables of their diff eqn (or invert) & integrate each side, increasing powers by 1 (or $\frac{1}{r} \rightarrow \ln r$)	*M1	their d.e. must be $\frac{dr}{dt}$ (or $\frac{dt}{dr}$) = f(r)	Ignore absence of '+c' after integration
			Subst $\frac{dr}{dt} = 1.08, r = 9$ into their diff eqn to find k	M1	their d.e. must include $\frac{dr}{dt}$ (or $\frac{dt}{dr}$), r & k	(✓k = 3.24 but M mark, not A)
			Substitute t = 5, r = 9 to find 'c'	dep*M1	Must involve '+c' here	
			Correct value of c (probably = 1.8 or -1.8)	A1	Check other values	
			$r = (4.86t + 2.7)^{\frac{2}{3}}$ ISW	A1	Answer required in form $r = f(t)$	
				[7]		
8	(ii)		subst t = 0 into any version of (i) result to find finite r	M1		(✓$r \approx 1.938991\ldots$.but M mark, not A)
			Any V in range $30.5 \le V < 30.55$, but not fortuitously	A1	Accept 9.72π or $\frac{243}{25}\pi$	
				[2]		

Question			Answer	Marks	Guidance	
9	(i)		$\frac{dy}{dt} = 2(+)-\frac{2}{t^3}$; $\frac{dx}{dt} = -\frac{1}{t^2}$ oe soi ISW	B1, B1		
			$\frac{2}{t} - 2t^2$ or $-2\left(t^2 - \frac{1}{t}\right), \frac{2t^3-2}{-t}, -t^2\left(2-\frac{2}{t^3}\right)$ oe	B1	ISW. Must not involve (implied)'triple-deckers' e.g. fractions with neg powers…	… e.g. $\frac{2-2t^{-3}}{-t^2}$
				[3]		
9	(ii)		(Any of their expressions for $\frac{dy}{dx}$) = 0 or their $\frac{dy}{dt} = 0$	M1		
			$t = 1 \rightarrow$ (stationary point) = (0, 3)	A1	Not awarded if $\frac{dy}{dx}$ is wrong in (i) and used here BUT allow recovery if only explicitly considering $\frac{dy}{dt} = 0$	
			Consider values of x on each side of their critical value of x which lead to finite values of $\frac{dy}{dx}$	M1		
			Hence (0, 3) is a minimum point www	A1	Totally satis; values of x must be close to 0 & not going below or equal to $x = -1$	
				[4]		
9	(iii)		Attempt to find t from $x = \frac{1}{t} - 1$ and substitute into the equation for y	M1		
			$y = \frac{2}{x+1} + (x+1)^2$ oe (can be unsimplified) ISW	A1		
				[2]		

Question			Answer	Marks	Guidance	
10	(i)		$(1-x)^{-3} = 1 + -3.-x + \frac{-3.-4}{2}(-x)^2 + \ldots$ oe; accept $3x$ for $-3.-x$ &/or $-x^2$ or $(x)^2$ for $(-x)^2$ multiplication by x to produce **AG** (**A**nswer **G**iven)	M1 A1 **[2]**	As result is given, this expansion must be shown and then simplified. It must not just be stated as $1+3x+6x^2+\ldots$	For alternative methods such as expanding $(1-x)^3$ and multiplying by $x+3x^2+6x^3$ <u>or</u> using long division, consult TL
10	(ii)		Clear indication that $x = 0.1$ is to be substituted (estimated value is) $0.1+3(0.1)^2+6(0.1)^3 = \underline{0.136}$	M1 A1 **[2]**	e.g. $0.1+3(0.1)^2+6(0.1)^3$ stated	Calculator value $\rightarrow$ M0 (0.13717… is calculator value of $\frac{100}{729}$)
10	(iii)		Sight of $1-x = x\left(\frac{1}{x}-1\right)$ or $1-x = -x\left(1-\frac{1}{x}\right)$ or $\left(\frac{1}{x}-1\right)^3 = -\left(1-\frac{1}{x}\right)^3$ or $(\frac{1}{x}-1)^{-3} = -\left(1-\frac{1}{x}\right)^{-3}$ or $\left(\frac{1}{x}-1\right)^{-3} = -\left(1-\frac{1}{x}\right)^{-3}$ or equivalent Complete satisfactory explanation (no reference to style) www $[1+(-3)(-\frac{1}{x})+\frac{(-3)(-4)}{2}\left(-\frac{1}{x}\right)^2+\ldots]$ $\rightarrow -\frac{1}{x^2}-\frac{3}{x^3}-\frac{6}{x^4}$	B1 B1 M1 A1 **[4]**	(Answer Given) Simplified expansion may be quoted – it may have come from result in part (i). Answer for this expansion is not **AG**.	

Question			Answer	Marks	Guidance	
10	(iv)		Must say "Not suitable" and one of following: Either: requires $\left\|\frac{1}{x}\right\| < 1$, which is not true if $x = 0.1$ Or: substitution of positive/small value of x in the expansion gives a negative/large value (which cannot be an approximation to 100/729).	B1 **[1]**	<u>This B1 is dep on $x = 0.1$ used in (ii).</u> Or "because $\frac{1}{x} > 1$" Or "it gives -63100"	Realistic reason If choice given, do not ignore incorrect comments, but ignore irrelevant/unhelpful ones